Dynamic Spectrum Management in Cognitive Radio Networks

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About Presenter

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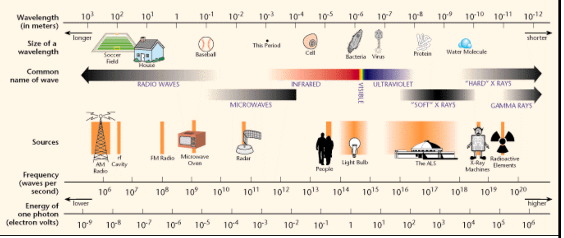
Tutorial Outline

- Static Spectrum Allocation
- Introduction to cognitive radio (CR)
- Dynamic Spectrum Access (DSA)/Management Models
- Dynamic Spectrum Access/Management Architecture
- Medium Access Control (MAC) for DSA
- Spectrum Trading: The Economics of Dynamic Spectrum Management
- Market-Equilibrium Pricing for Spectrum Sharing
- Game Theory for Dynamic Spectrum Access/Management
- Application of Oligopoly Models for Spectrum Sharing and Pricing
- Auction Theory for Dynamic Spectrum Access/Management
- Concluding Remarks
- References

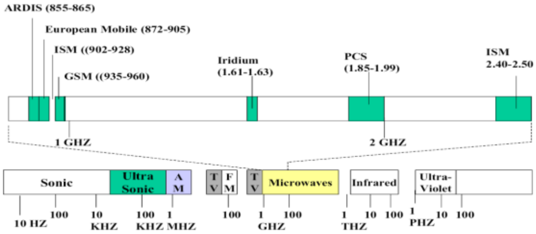
Static Spectrum Allocation

- What is it?
- What are the problems?
- Towards Dynamic Spectrum Access

Electro Magnetic Spectrum

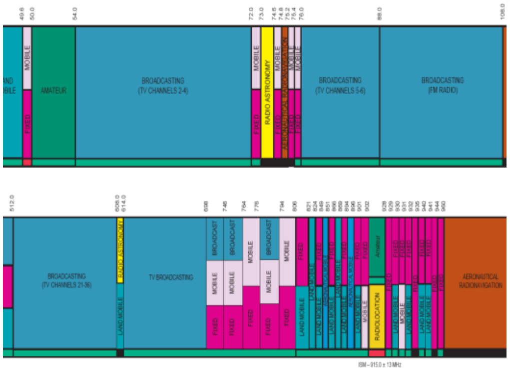


Electro Magnetic Spectrum



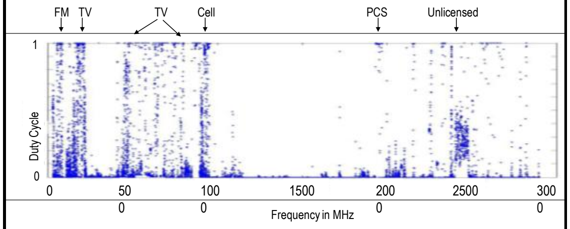
Static Spectrum Allocation

- Regulatory bodies allocate spectrum licenses to wireless service providers (WSPs).
  - ☐ Example: Federal Communications Commission (FCC) in USA.
  - ☐ Example: Telecom Regulatory of India (TRAI)
- These allocations are now usually done via auctions.
  - ☐ 33 auctions from 07/1994 - 02/2001 raised $40 billion in US.
  - ☐ 3G wireless license auction raised $100 billion in Europe.
- Licenses are granted on a long term basis to WSPs.
  - ☐ Licenses specify the range of frequencies to be used in particular geographic areas.
  - ☐ Restrictions are imposed on the technologies to be used and the services to be provided.

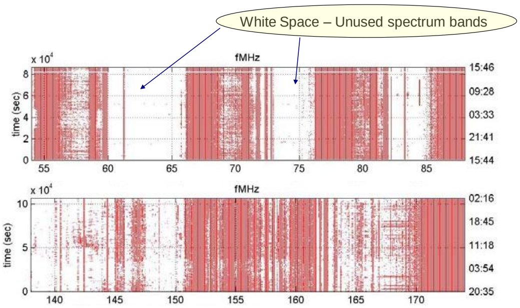


NYC Occupancy Summary

- 16% Duty Cycle – All Bands 30 MHz to 3000 MHz, 24 hour period
- Peak Usage Period – Political Convention (2004)



Sample Results



Spectrum usage measured at different time over different spectrum bands

Disadvantages of Static Spectrum Allocation

- Portions of the spectrum are largely unoccupied most of the time.
- Spectrum allocations are fixed, long–term
  - ☐ any changes are made under the strict guidance of government
- Demand of spectrum in cellular networks is higher than anticipated
- Large parts of spectrum allocated to the military, public safety systems
  - ☐ These bands are highly under utilized
  - ☐ Spectrum holes exist in electromagnetic spectrum
- Modification of old technologies
  - ☐ Example: Digital TV broadcasts require 50% of analog versions
  - ☐ IEEE 802.22 (Cognitive radio based WRAN standard)

Inference

- Increase in spectrum demand due to new wireless technologies and applications (spectrum scarcity)
- Spectrum access a more significant problem than spectrum scarcity - even in crowded areas, more than half of the spectrum unused (depending on time and location)

Spectrum utilization can be significantly improved by ***allowing "unlicensed" users to borrow <u>idle</u> spectrum from "spectrum licensees"*** **–**

- A phenomenon known as **Dynamic Spectrum Access**

- Spectrum Licensees → Primary User/Node
- Unlicensed User → Secondary User/Node

Introduction to Cognitive Radio

- Definition
- Key Features/Capabilities
- Primary Tasks and CR Network Architecture
- Next Generation Wireless Networks

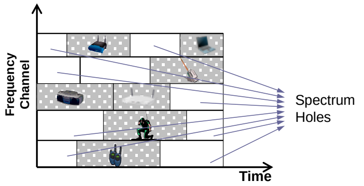
Motivation

- FCC (Federal Communications Commission) Nov.'02 Spectrum Policy Task Force Report [SCJ1]:
  - some bands are largely unoccupied (white spaces)
  - some are partially occupied (grey spaces)
  - some are heavily used (black spaces).
- Field measurement:
  - total spectrum occupancy in the band 30 MHz – 3 GHz is only 13.1% (in New York city) [SCJ1]
- Spectrum scarcity and spectrum underutilization (motivations for cognitive radio)

Cognitive Radio

- "Cognitive radio" – a novel way to improve utilization of electromagnetic radio spectrum (solve spectrum underutilization problem)
- Definition [F14, F15]: An intelligent wireless communication system *aware* of the environment which uses some methodology to *learn* from the environment and *adapt* to statistical variations in the environment (input RF stimuli) by adapting the protocol parameters (e.g., transmit power, carrier frequency, and modulation strategy at the physical layer) in an online fashion.
- Ojgective: Achieve highly reliable communication and efficient utilization of radio spectrum (by utilizing *spectrum holes*).

Spectrum Hole/Opportunity



- *Spectrum hole*: band of frequencies assigned to a licensed (primary) user but not utilized at a particular time and location.
- Spectrum hole can be identified in these dimensions
  - Frequency, Time, Location, Power

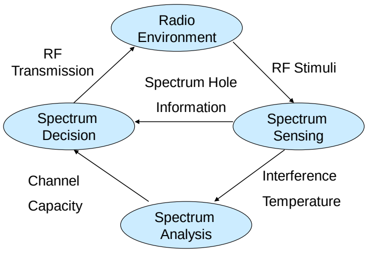
Key Features/Capabilities

- Awareness (RF spectrum, transmitted waveform, wireless network protocols, services, and applications)
- Intelligence (through learning, internal tuning of parameters through communication using cognitive language)
- CR networks/NeXt Generation (xG) networks/dynamic spectrum access (DSA) networks [F16]

Basic Cognitive Tasks

- Spectrum Sensing (detection of spectrum holes, estimation of channel state information, traffic statistics, other protocol/system parameters)
- Spectrum Analysis (detection of the characteristics of spectrum holes, channel capacity, predictive modeling)
- Spectrum Access Decision (transmit or not to transmit, select appropriate transmission band, adapt transmission parameters – e.g., transmit power, data rate, modulation level)

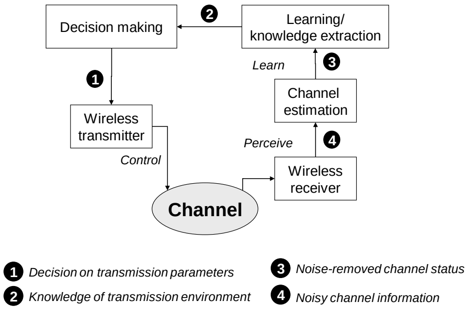
Cognitive Cycle [F14,F15]



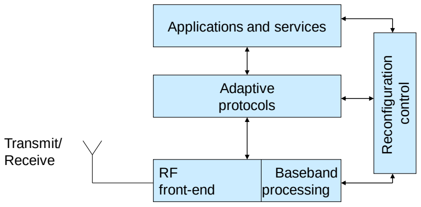
Primary Tasks of a CR Network

- Perceive radio environment
- Learn from the environment and adapt radio transceiver performance
- Facilitate communication among multiple users through cooperation
- Control communication processes among competing users through proper radio resource allocation

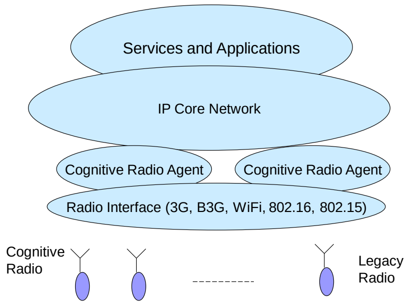
Primary Tasks of a CR Network

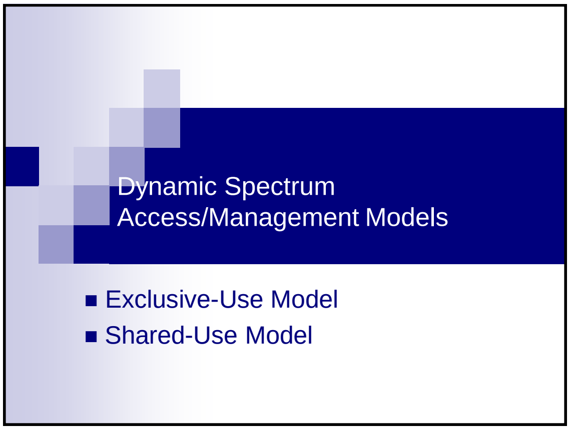


CR Architecture



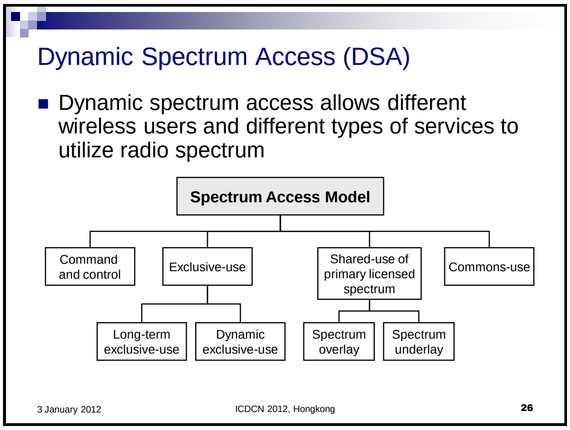
Next Generation Wireless Networks





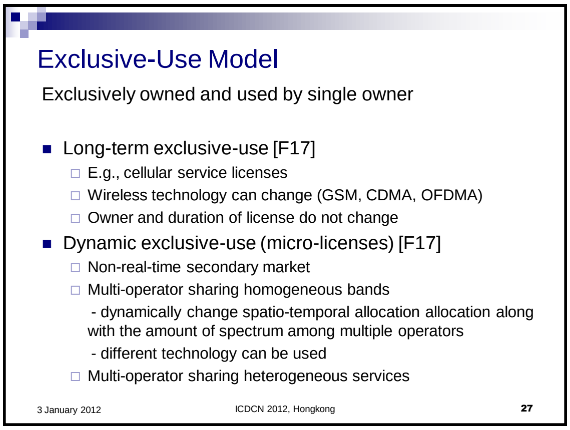
Dynamic Spectrum Access/Management Models

- Exclusive-Use Model
- Shared-Use Model



Dynamic Spectrum Access (DSA)

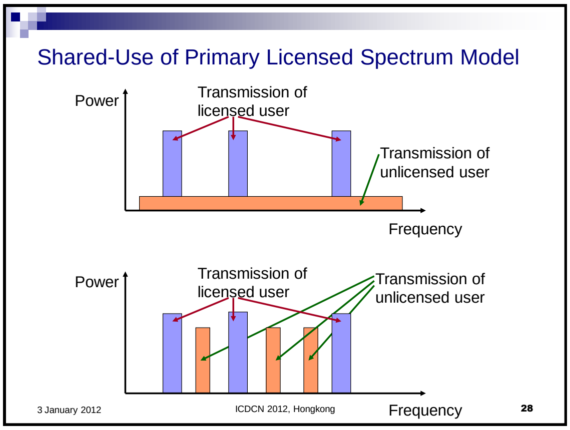
- Dynamic spectrum access allows different wireless users and different types of services to utilize radio spectrum



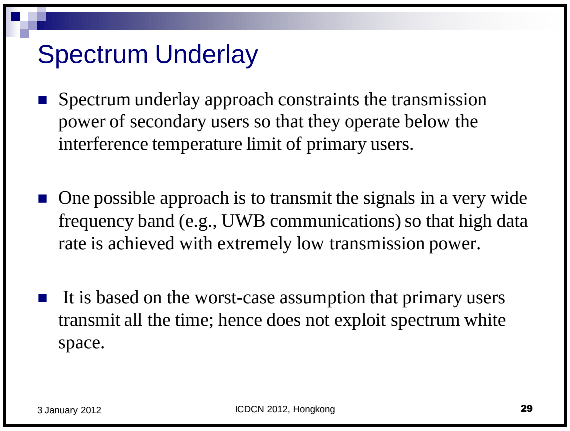
Exclusive-Use Model

Exclusively owned and used by single owner

- Long-term exclusive-use [F17]
  - E.g., cellular service licenses
  - Wireless technology can change (GSM, CDMA, OFDMA)
  - Owner and duration of license do not change
- Dynamic exclusive-use (micro-licenses) [F17]
  - Non-real-time secondary market
  - Multi-operator sharing homogeneous bands
    - dynamically change spatio-temporal allocation allocation along with the amount of spectrum among multiple operators
    - different technology can be used
  - Multi-operator sharing heterogeneous services

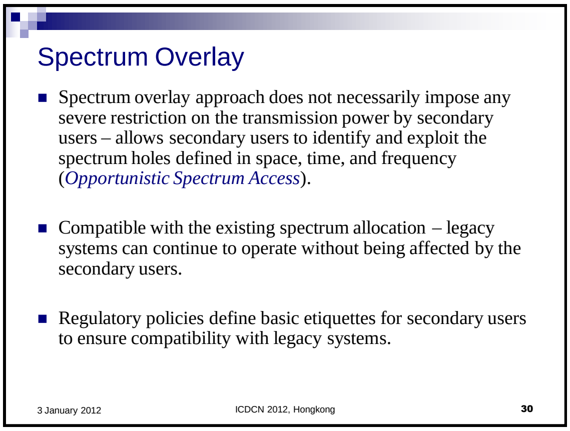


Shared-Use of Primary Licensed Spectrum Model



Spectrum Underlay

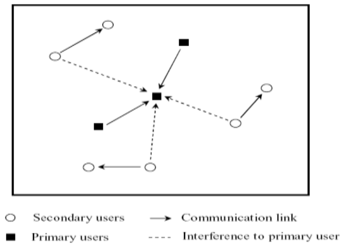
- Spectrum underlay approach constraints the transmission power of secondary users so that they operate below the interference temperature limit of primary users.
- One possible approach is to transmit the signals in a very wide frequency band (e.g., UWB communications) so that high data rate is achieved with extremely low transmission power.
- It is based on the worst-case assumption that primary users transmit all the time; hence does not exploit spectrum white space.



Spectrum Overlay

- Spectrum overlay approach does not necessarily impose any severe restriction on the transmission power by secondary users – allows secondary users to identify and exploit the spectrum holes defined in space, time, and frequency (*Opportunistic Spectrum Access*).
- Compatible with the existing spectrum allocation – legacy systems can continue to operate without being affected by the secondary users.
- Regulatory policies define basic etiquettes for secondary users to ensure compatibility with legacy systems.

Hybrid of Spectrum Overlay and Spectrum Underlay



Spectrum Access/Management Architecture

- Cooperative and Non-cooperative Spectrum Access
- Centralized and Distributed Spectrum Management

Spectrum Management Issues

- How to share among secondary users (MAC issue)
- Local optimization (decision is taken in a non-cooperative way - distributed)
- Global optimization (cooperative decision – centralized or distributed)
- How to communicate spectrum access decisions among cognitive radio transmitters and receivers

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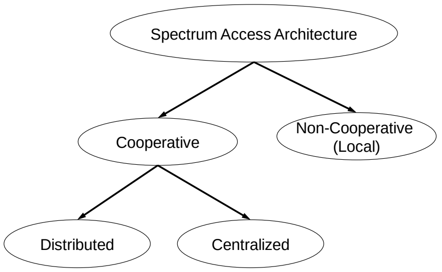
DSA Architecture



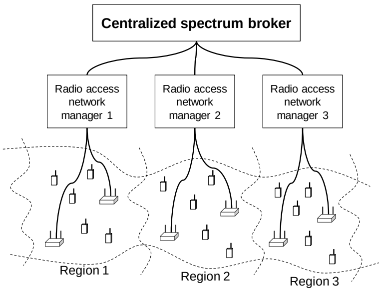
Non-cooperative Spectrum Access

- Each cognitive node is responsible for its own decision.
- E.g., if miss detection probability is large, access policy should be conservative. If false alarm probability is large, access policy should be aggressive
  - access strategy can be jointly optimized with the sensing strategy (MAC design issue)
- Minimal communication requirements (hence less overhead), but spectrum utilization may be poor.

Cooperative/Centralized DSA

- A centralized server maintains a database of spectrum availability and access information (based on information received from secondary users, e.g., through a dedicated control channel).
- Spectrum management is simpler and coordinated and enables efficient spectrum sharing.

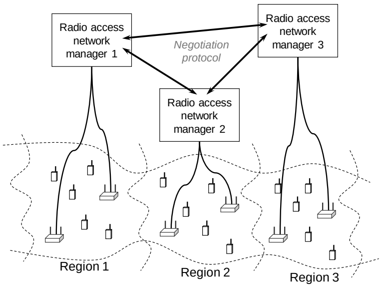
Cooperative/Centralized DSA



Cooperative/Distributed DSA

- Cooperative/distributed strategy relies on cooperative local actions throughout the network (to achieve a performance close to the global optimal performance).
- May suffer due to hidden node problem and large control overheads
- In both centralized and distributed strategies, the primary user may or may not cooperate.

Cooperative/Distributed DSA



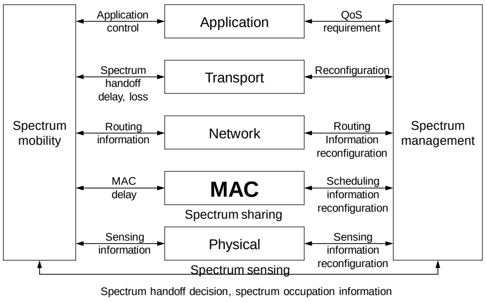
Spectrum Access/Management Challenges

- Unavailability of any fixed common control channel (availability changes depending on the spectrum access by primary users)
- Local optimization strategies (e.g., learning-based algorithms)
- Spectrum pricing models for DSA

Medium Access Control (MAC) for DSA

- CR Protocol Stack and MAC
- MAC Functions and Challenges

Protocol Stack in DSA Networks [F16]



Spectrum handoff decision, spectrum occupation information

MAC Design Issues for DSA

- Decision on whether to transmit (spectrum sensing/detection could be erroneous, is the price offered/charged by the primary user/service provider acceptable?)

- How to exploit the spectrum holes (what modulation and power level to use, how to share the spectrum holes among cognitive radios - centralized/distributed, competitive/cooperative).

- Spectrum access decisions may need to be communicated among the cognitive nodes and/or the intended receiver.

MAC for DSA

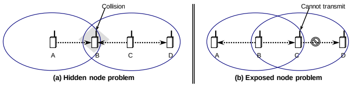
- MAC functions
  - ☐ Obtain information on channel occupancy (spectrum sensing) and make decision on spectrum access
  - ☐ Synchronize transmission parameters (e.g., channel, time slot) between transmitter and receiver
  - ☐ Facilitate negotiation among primary users and secondary users for spectrum allocation
  - ☐ Facilitate communication among secondary users to perform channel sensing and channel access
  - ☐ Facilitate spectrum trading functions (e.g., spectrum bidding and spectrum pricing)

MAC for DSA

- Major Challenges
  - ☐ Optimal channel sensing and channel access
  - ☐ Synchronization between transmitter and receiver
  - ☐ Optimal channel allocation/scheduling, rate and power adaptation
  - ☐ Coexistence between primary and secondary users
  - ☐ Primary user's time-varying activity
  - ☐ Hidden and exposed terminal problems
  - ☐ Multi-channel access
  - ☐ Analytical models

MAC for DSA

- Hidden node and exposed node problems



(a) Hidden node problem

(b) Exposed node problem

MAC for DSA

- State of the Art Approaches
  - ☐ MAC to optimize spectrum sensing and spectrum access (distributed)
  - ☐ MAC for multi-channel and multi-user access (distributed)
  - ☐ MAC based on spectrum and power allocation/scheduling (centralized)
  - ☐ MAC to support spectrum trading (centralized and distributed)

MAC for DSA: Example Protocols

- ☐ Cognitive MAC Protocol for Joint Spectrum Sensing and Spectrum Access Optimization [MAC2]
- ☐ Hardware-Constrained Multi-Channel Cognitive MAC (HC-MAC) protocol [MAC3]
- ☐ MAC protocol for efficient discovery of spectrum opportunities [MAC9]
- ☐ Opportunistic Spectrum Access MAC (OSA-MAC) [MAC10]
- ☐ Ad Hoc Secondary System MAC (AS-MAC) for Spectrum Sharing [MAC1]
- ☐ Dynamic frequency hopping MAC [MAC5]
- ☐ Rate and Power Adaptation MAC [MAC8]
- ☐ Rental Protocol for Dynamic Channel Allocation [MAC11]

MAC for DSA

- Open issues
  - ☐ Consider PHY layer specifics (i.e., cross-layer optimized MAC protocols)
  - ☐ Customized MAC protocols for specific applications (e.g., ITS, health-care applications)
  - ☐ Reduce communication overhead in exchanging information (when the number of secondary users is large or traffic load is heavy, the control channel becomes the bottleneck)
  - ☐ Optimize MAC protocols from “Economics” perspective

Spectrum Trading:
The Economics of Dynamic Spectrum Access/Management

- Definition
- Issues in Spectrum Trading
- Approaches in Spectrum Trading

Spectrum Trading - Definition

- **Trading** is defined as the process of exchanging goods or services in a market
- Exchange can be performed directly between goods and services (i.e., bartering), or by using a medium of exchange (e.g., money)
- **Spectrum trading** is the process of selling and buying radio spectrum between primary users (service providers) and secondary users/service providers
- Generate revenue for spectrum owner while providing satisfaction to cognitive radio users

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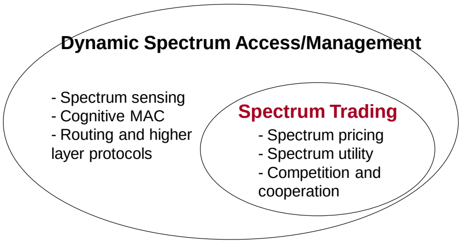
Spectrum Trading - Definition



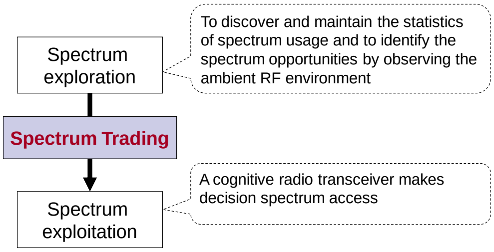
Dynamic Spectrum Access/Management

- Spectrum sensing
- Cognitive MAC
- Routing and higher layer protocols

Spectrum Trading

- Spectrum pricing
- Spectrum utility
- Competition and cooperation

Spectrum Trading - Definition



Spectrum exploration

To discover and maintain the statistics of spectrum usage and to identify the spectrum opportunities by observing the ambient RF environment

Spectrum Trading

Spectrum exploitation

A cognitive radio transceiver makes decision spectrum access

Structure of Spectrum Trading

- Single Seller – Monopoly
  - ☐ A single seller could be spectrum owner in exclusive usage and primary user in private commons model
- Multiple Sellers – Oligopoly
  - ☐ Multiple spectrum owners and multiple primary users
- No Permanent Seller - Exchange Market
  - ☐ All users have a right to access the spectrum as in public commons model

Issues in Spectrum Trading

- Spectrum Pricing
  - ☐ Satisfaction of spectrum access (for secondary users) and revenue (for primary users) depend on spectrum price
- Spectrum Supply and Cost of Spectrum Sharing
  - ☐ Spectrum supply function gives the amount of bandwidth available for the spectrum buyers

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Issues in Spectrum Trading

- Utility Function and Spectrum Demand
  - ☐ Spectrum demand determines the amount of spectrum that the buyer (i.e., cognitive radio) wants to access for a given price so that utility is maximized
- Competition and Cooperation in Spectrum Sharing
  - ☐ Sellers/Buyers can compete or cooperate with each other to achieve their objectives

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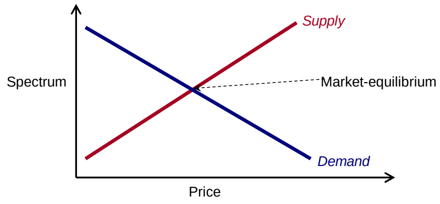
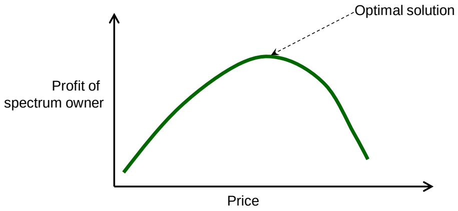
Approaches in Spectrum Trading

- Microeconomic Approach
  - ☐ Market-equilibrium is the point where spectrum demand equals to spectrum supply

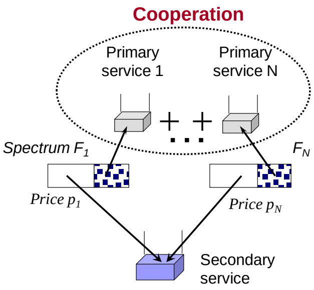


Approaches in Spectrum Trading

- Classical Optimization Approach
  - ☐ Optimal solution to maximize revenue or utility

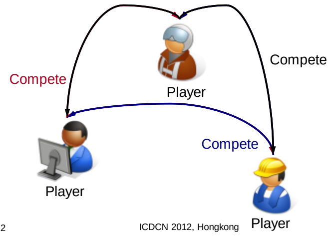


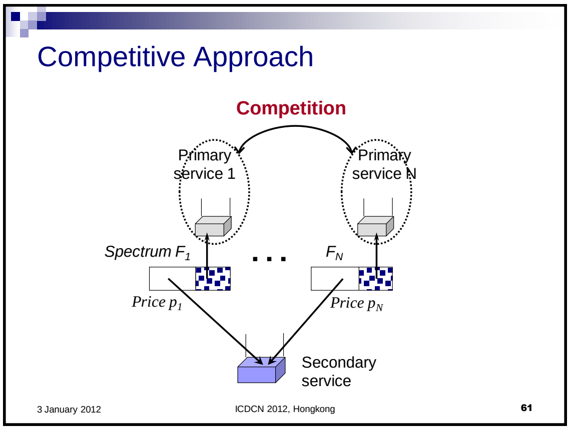
Cooperative Approach



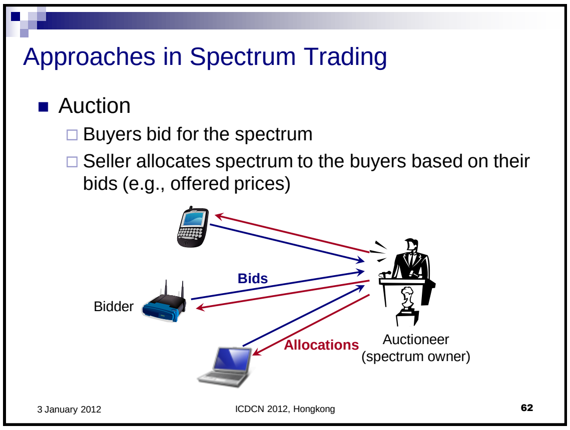
Approaches in Spectrum Trading

- Noncooperative Game
  - ☐ Nash equilibrium solution satisfies all entities in spectrum trading and individual payoff is maximized



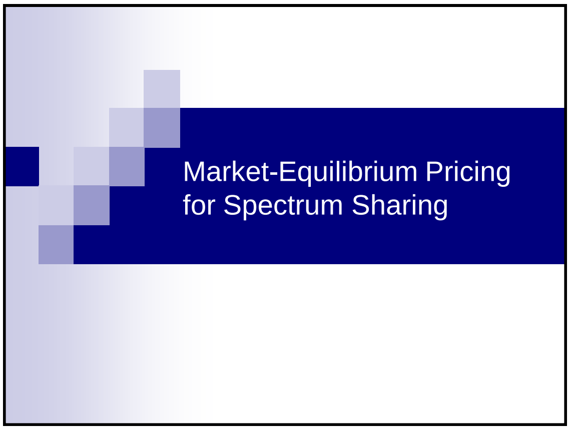


Competitive Approach

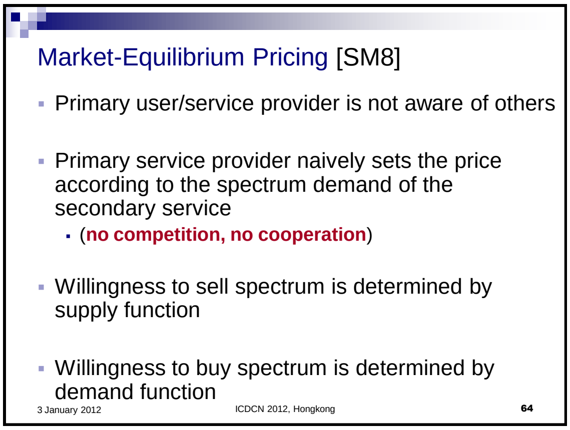


Approaches in Spectrum Trading

- Auction
  - ☐ Buyers bid for the spectrum
  - ☐ Seller allocates spectrum to the buyers based on their bids (e.g., offered prices)

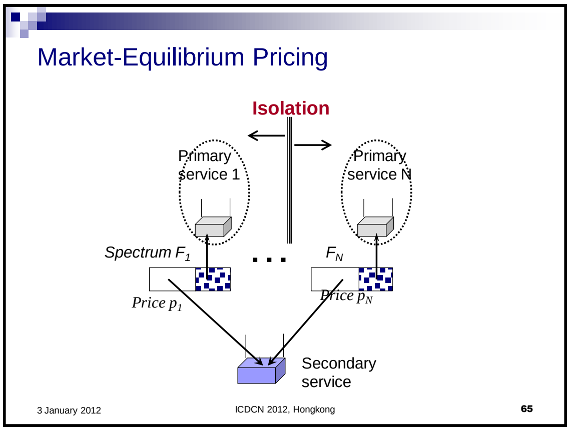


Market-Equilibrium Pricing for Spectrum Sharing

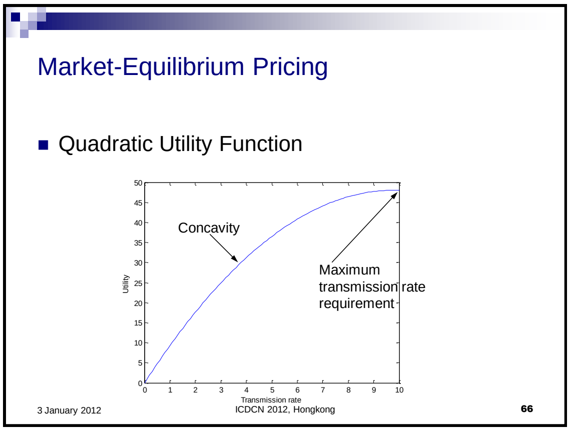


Market-Equilibrium Pricing [SM8]

- Primary user/service provider is not aware of others
- Primary service provider naively sets the price according to the spectrum demand of the secondary service
  - (**no competition, no cooperation**)
- Willingness to sell spectrum is determined by supply function
- Willingness to buy spectrum is determined by demand function



Market-Equilibrium Pricing



Market-Equilibrium Pricing

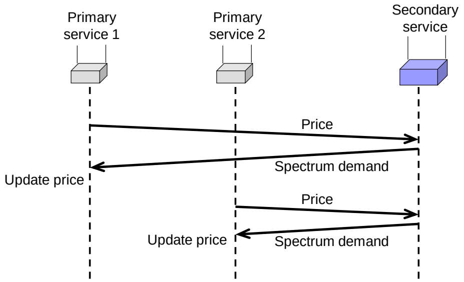
- Quadratic Utility Function

Market-Equilibrium Pricing

- Spectrum demand - e.g., a linear function of prices of all primary service providers
- Two sources of revenue for primary
  - Primary users (fixed price – discount due to performance degradation)
  - Secondary users
- Spectrum supply - obtained by taking derivative of revenue w.r.t. spectrum size
- Market-Equilibrium: Spectrum Demand = Spectrum Supply

Market-Equilibrium Pricing

- Distributed implementation [SM8]



Game Theory for Design and Analysis of Cognitive Radio Networks

- Introduction
- Static and Dynamic Games
- Normal Form and Extensive Form Games
- Nash Equilibrium and Best Response

Introduction

- **Game theory** – mathematical models and techniques developed in economics to analyze interactive decision processes, predict the outcomes of interactions, identify optimal strategies [SM30]
- Game theory techniques were adopted to solve many protocol design issues (e.g., resource allocation, power control, cooperation enforcement) in wireless networks.
- Fundamental component of game theory is the notion of a *game*.

Introduction

- A game is described by a set of rational *players*, the *strategies* associated with the players, and the *payoff*s for the players.
  - A rational player has his own interest, and therefore, will act by choosing an available strategy to achieve his interest.
- A player is assumed to be able to evaluate exactly or probabilistically the outcome or payoff (usually measured by the utility) of the game which depends not only on his action but also on other players' actions.

Static and Dynamic Games

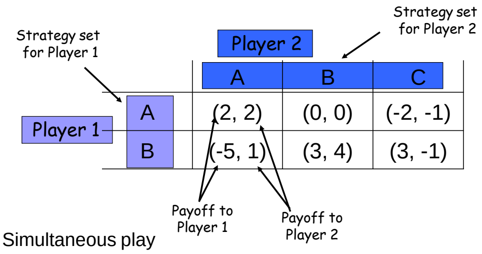
- In static games (one-shot games), the players make their moves in isolation without knowing what other players have done. This does not necessarily mean that all decisions are made at the same time, but rather only as if the decisions were made at the same time.
- Dynamic games have a sequence to the order of play and players observe some, if not all, of one another's moves as the game progresses.
- In non-cooperative game theory there are two ways in which a game can be represented – normal form game or strategic form game and extensive form game.

Games in Strategic (Normal) Form

- **Players**: finite set of players,
  - □{1, 2, … , N}
- **Strategy Space**: formed from the Cartesian product of each player's strategy set,
  - □$A = A_1 \times A_2 \times \ldots \times A_N$
- **Payoffs**: set of utility functions,
  - □$\{u_1, u_2, \ldots , u_N\}$

Matrix Representation of Game

- Representation of a game



	A	B	C
A	(2, 2)	(0, 0)	(-2, -1)
B	(-5, 1)	(3, 4)	(3, -1)

- Simultaneous play
  - □ players analyze the game and write their strategy on a paper
- Combination of strategies determines payoff

Example: The Prisoner's Dilemma

Players: Two suspects

Strategies: Each player's set of strategies is {Quiet, Confess}

Payoffs:

u_1 (Confess, Quiet) = 0, u_1 (Quiet, Quiet) = -2

u_2 (Quiet, Confess) = -10, u_2 (Quiet, Quiet) = -2

Example: The Prisoner's Dilemma

Suspect 2

Suspect 1		Confess	Quiet
	Confess	-5,-5	0,-10
	Quiet	-10,0	-2,-2

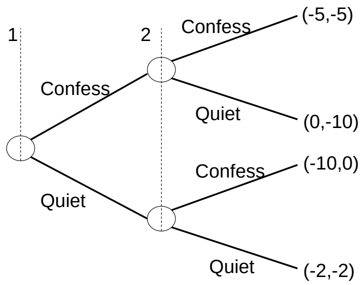
Games in Extensive Form

- This type of game is represented with a game tree. Extensive form games have the following four elements in common:
- Nodes: This is a position in the game where one of the players must make a decision. The first position, called the initial node, is an open dot, all the rest are filled in. Each node is labeled so as to identify who is making the decision.
- Branches: These represent the alternative choices that the player faces, and so correspond to available actions.

Games in Extensive Form

- Vectors: These represent the pay-offs for each player, with the pay-offs listed in the order of players.
- When these payoff vectors are common knowledge the game is said to be one of complete information.
- If, however, players are unsure of the pay-offs other players can receive, then it is an incomplete information game.
- Information sets: When two or more nodes are joined together by a dashed line this means that the player whose decision it is does not know which node he or she is at. When this occurs the game is characterized as one of imperfect information.
- When each decision node is its own information set the game is said to be one of perfect information, as all players know the outcome of previous decisions.

Example: The Prisoner's Dilemma



Games in Extensive Form

- While the normal form gives the minimum amount of information necessary to describe a game, the extensive form gives additional details about the game concerning the timing of the decisions to be made and the amount of information available to each player when each decision has to be made.
- For every extensive form game, there is one and only one corresponding normal form game. For every normal form game, there are, in general, several corresponding extensive form games.

Static Games in Normal Form

- Static games are predominantly represented as normal form games.
- Because in such games the amount of information available to players does not vary within the game, and the timing of decisions has no effect on players' choices.

Solving Static Games

- **Notations [SM31]**

Let **a** be an action profile in which the action of each player **i** is $\mathbf{a}_i$. Let $\mathbf{a}_i'$ be any action of player **i** (either equal to $\mathbf{a}_i$ or different from it). Then ($\mathbf{a}_i'$, $\mathbf{a}_{-i}$) denotes the action profile in which all the players other than **i** adhere to **a** while **i** "deviates" to $\mathbf{a}_i'$.

For example, in a 3-player game, ($\mathbf{a}_2'$, $\mathbf{a}_{-2}$) denotes the action profile in which players 1 and 3 adhere to **a** while player 2 deviates to $\mathbf{a}_2'$.

Nash Equilibrium

- A Nash equilibrium is an action profile **a*** with the property that no player **i** can do better by choosing an action different from $\mathbf{a}_i^*$, given that every other player **j** adheres to $\mathbf{a}_j^*$.
- That is, for every player i,

$$\mathbf{u_i(a^*) \geq u_i\ (a_i, a_{-i}^*)}$$

for every action $\mathbf{a}_i$ of player **i**, where $\mathbf{u}_i$ is the payoff function for player **i**.

- A Nash equilibrium corresponds to a steady state of the game among "experienced players". It represents an outcome that results from the simultaneous maximization of individual payoffs.

Nash Equilibrium

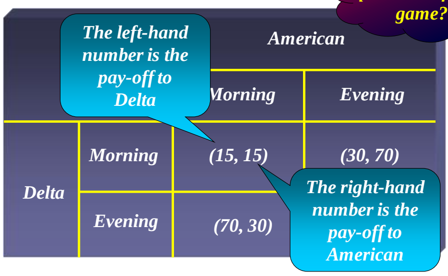
- If a Nash equilibrium is common knowledge, then every player would indeed play the Nash equilibrium strategy, thereby resulting in the Nash equilibrium being played.
- In other words, a NE strategy profile is self-enforcing. If the players are searching for outcomes or solutions from which no player will have an incentive to deviate, then the only strategy profiles that satisfy such a requirement are the Nash equilibria.

An example (Sequential elimination)

- Two airlines set the prices for tickets.
- Compete for departure times
- 70% of consumers prefer evening departure, 30% prefer morning departure
- If the airlines choose the same departure times they share the market equally
- Pay-offs to the airlines are determined by market shares
- Represent the pay-offs in a ***pay-off matrix***

The Payoff Matrix

What is the equilibrium for this game?

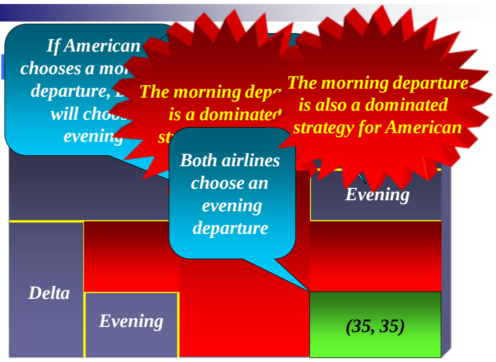


		American	
		Morning	*Evening*
Delta	*Morning*	*(15, 15)*	*(30, 70)*
	Evening	*(70, 30)*	

The left-hand number is the pay-off to Delta

The right-hand number is the pay-off to American

Th



If American chooses a mo departure, will choo evening

The morning depa is a dominated st

The morning departure is also a dominated strategy for American

Both airlines choose an evening departure

Example of NE

- **Prisoner's Dilemma**

Players: Two suspects

Strategies: Each player's set of strategies is {Quiet, Confess}

Payoffs: e.g.,

u_1 (Confess, Quiet) = 0, u_1 (Quiet, Quiet) = -2, …

u_2 (Quiet, Confess) = -10, u_2 (Quiet, Quiet) = -2, …

Player 1 prefers (C, Q) to (Q, Q) to (C, C) to (Q, C)

Player 2 prefers (Q, C) to (Q, Q) to (C, C) to (C, Q)

Example of NE: Prisoner's Dilemma in Strategic Form

Nash Equilibrium

		Suspect 2	
		Confess	Quiet
Suspect 1	Confess	-5,-5	0,-10
	Quiet	-10,0	-2,-2

Better outcome

Example of NE

- The action profile (**Confess**, **Confess**) is the only NE.
- To show that a pair of actions is not a Nash equilibrium, it is enough to show that one player wishes to deviate (an equilibrium is immune to any unilateral deviation).
- In general, at the Nash equilibrium, the action for a player is optimal if the other players choose their Nash equilibrium actions, but some other action is optimal if the other players choose non-equilibrium actions.

Mixed Strategy Nash Equilibrium

- A mixed strategy is when a player randomizes over some or all of his or her available pure strategies. That is, the player places a probability distribution over their alternative strategies.
- A mixed-strategy equilibrium is where at least one player plays a mixed strategy and no one has the incentive to deviate unilaterally from that position.
- Every matrix game has a Nash equilibrium in mixed strategies.
- Every NE in pure strategies is also a NE of the game in mixed strategies.

Best Response Functions

- For any given actions of the players other than i, the best actions of player i which yield the highest payoff for player i, denoted by, $\mathbf{B}_i\,(\mathbf{a}_{-i})$
- $\mathbf{B}_i$ = best response function of player **i**.
- Mathematically,

$\mathbf{B}_i\,(\mathbf{a}_{-i}) = \{\mathbf{a}_i \text{ in } \mathbf{A}_i\text{: } \mathbf{u}_i(\mathbf{a}_i, \mathbf{a}_{-i}) \geq \mathbf{u}_i(\mathbf{a}_i', \mathbf{a}_{-i}) \text{ for all } \mathbf{a}_i' \text{ in } \mathbf{A}_i\}$,

i.e., any action in $\mathbf{B}_i\,(\mathbf{a}_{-i})$ is at least as good for player i as every other action of player i when the other players' actions is given by $\mathbf{a}_{-i}$.

Best Response Functions

- NE is an action profile for which every player's action is a best response to the other players' actions, that is, the action profile **a*** is a Nash equilibrium if $\mathbf{a}_i^*$ is in $\mathbf{B}_i(\mathbf{a}_{-i}^*)$ for every player **i**.
- Example:

In a two-player game in which each player has a single best response to every action of the other player, $(\mathbf{a}_1^*, \mathbf{a}_2^*)$ is a NE iff player 1's action $\mathbf{a}_1^*$ is his/her best response to player 2's action $\mathbf{a}_2^*$, and player 2's action $\mathbf{a}_2^*$ is his/her best response to player 1's action $\mathbf{a}_1^*$.

Best Responses in Prisoner's Dilemma

- BR of Suspect 1 to each action of Suspect 2:
  S2 chooses C → BR of S1 is C (i.e., (C, C))
  S2 chooses Q → BR of S1 is C (i.e., (C, Q))
- BR of Suspect 2 to each action of Suspect 1:
  S1 chooses C → BR of S2 is C (i.e., (C, C))
  S1 chooses Q → BR of S2 is C (i.e., (Q, C))
- The game has one NE: (C, C)

	Suspect 2		
Suspect 1		Confess	Quiet
	Confess	(-5, -5)**	(0, -10)
	Quiet	(-10, 0)*	(-2, -2)*

Game Models for Dynamic Spectrum Sharing and Pricing

- Cournot's Oligopoly Game
- Bertrand's Oligopoly Model
- Stackelberg's Model of Duopoly

Cournot's Oligopoly Game [SM30]

- To model interactions between firms competing for the business of consumers (oligopoly means "competition between a small number of sellers").
- A single good is produced by **n** firms. The cost to firm **i** of producing $\mathbf{q}_i$ units of the good is $\mathbf{C}_i(\mathbf{q}_i)$, where $\mathbf{C}_i$ is an increasing function.
- If the firms' total output is **Q**, then the market price is **P(Q). P** is called the "inverse demand function".
- In Cournot model, the firms compete in terms of quantity supplied to the market.

Cournot's Game

- Assume that **P** is a decreasing function when it is positive: if the firms' total output increases, then the price decreases (unless it is already zero).
- If the output of each firm **i** is q_i, then the price is $P(q_1 + \dots + q_n)$, so that firm i's revenue is $q_i\, P(q_1 + \dots + q_n)$. Thus, firm **i**'s profit, equal to its revenue minus its cost, is $\pi_i(q_1, \dots, q_n) = q_i\, P(q_1 + \dots + q_n) - C_i(q_i)$
- Players: The firms

  Strategies: Each firm's set of strategies is the set of its possible outputs (nonnegative numbers)

  Payoffs: Each firm's payoffs are represented by its profit

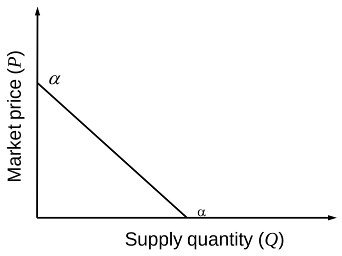
Example: Duopoly with Constant Unit Cost and Linear Inverse Demand Function

- There are two firms and $C_i(q_i) = cq_i$ for all q_i (here **c** is the unit cost). The inverse demand function is given by: $P(Q) = \alpha - Q$ if $Q \le \alpha$, and $P(Q) = 0$, if $Q > \alpha$, (where $\alpha > 0$ and $c \ge 0$ are constants).
- The NE can be obtained based on the firms' best response functions. If the firms' outputs are q_1 and q_2, then market price is,

$P(q_1 + q_2) = \alpha - q_1 - q_2$ if $q_1 + q_2 \le \alpha$

$P(q_1 + q_2) = 0$ if $q_1 + q_2 > \alpha$.

Inverse Demand Function



Cournot's Duopoly

- Firm 1's profit is,

$\pi_1(q_1, q_2) = q_1(P(q_1 + q_2) - c)$

$= q_1(\alpha - c - q_1 - q_2)$,

when $q_1 + q_2 \le \alpha$, and

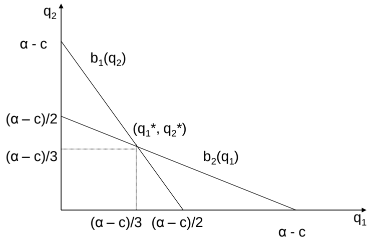
$\pi_1(q_1, q_2) = -cq_1$, when $q_1 + q_2 > \alpha$.

- The response function gives the strategy that maximizes this profit. Differentiating $\pi_1(q_1, q_2)$ with respect to q_1, $b_1(q_2) = ½ (\alpha - c - q_2)$. Similarly, $b_2(q_1) = ½ (\alpha - c - q_1)$.

Cournot's Duopoly

- A NE is a pair (q_1^*, q_2^*) of outputs for which q_1^* is the best response to q_2^*, and q_2^* is a best response to q_1^*: $q_1^* = b_1(q_2^*)$ and $q_2^* = b_2(q_1^*)$.
- Solving $b_1(q_2) = ½ (\alpha - c - q_2)$ and $b_2(q_1) = ½ (\alpha - c - q_1)$, $q_1^* = q_2^* = 1/3\ (\alpha - c)$.
- The total output at this equilibrium is **2/3 (α - c)** and the price is $P(2/3\ (\alpha - c)) = \alpha - 2/3(\alpha - c) = 1/3(\alpha + 2c)$.
- As **α** increases (i.e., consumers are willing to pay more), the equilibrium price and the output of each firm increases.
- As **c** (unit cost of production) increases, the output of each firm falls and the price rises.

Best Response Functions in Cournot's Duopoly



Bertrand's Oligopoly Model **[SM30]**

- In Cournot's game a firm changes its behavior if it can increase its profit by changing its output, on the assumption that other firms' output will remain the same and the price will adjust to clear the market.
- In Bertrand's game a firm changes its behavior if it can increase its profit by changing its price, on the assumption that the other firms' prices will remain the same and their outputs will adjust to clear the market.
- In both cases, each firm chooses its action not knowing the other firms' actions.

Bertrand's Game

- A single good is produced by **n** firms; each firm can produce $\mathbf{q_i}$ units of the good at a cost of $\mathbf{C_i(q_i)}$.
- The inverse demand function **D** gives the total amount of demand **D(p)** as a function of price **p**.

  $\mathbf{D(p) = \alpha - p}$ for $\mathbf{p \leq \alpha}$ and $\mathbf{D(p) = 0}$ for $\mathbf{p > \alpha}$
- If the firms set different prices, then all consumers purchase the good from the firm with the lowest price, which produces enough output to meet this demand.
- If more than one firm sets the lowest price, all the firms doing so share the demand at that price equally.

Bertrand's Game

- A firm whose price is not the lowest price receives no demand and produces no output.
- Note that a firm does not choose its output strategically; it simply produces enough to satisfy all the demand it faces, given the prices, even if its price is below its unit cost, in which case it makes a loss

Bertrand's Game

- Bertrand's oligopoly game is the following strategic game:

  Players: The firms

  Strategies: Each firm's set of strategies is the set of possible prices (nonnegative numbers)

  Payoffs: Firm i's payoff is represented by its profit $\mathbf{p_iD(p_i)/m - C_i(D(p_i)/m)}$ if firm **i** is one of m firms setting the lowest price (**m = 1** if firm i's price $\mathbf{p_i}$ is lower than every other price), and **m =** ∞ if some firm's price is lower than $\mathbf{p_i}$.

Example: Duopoly with Constant Unit Cost and Linear Demand Function

- Since firm **i** makes the profit of $\mathbf{p_i - c}$ on every unit, it's profit is

  $\mathbf{\pi_i (p_1, p_2) = (p_i - c) (\alpha - p_i)}$ **if** $\mathbf{p_i < p_j}$

  $\mathbf{\pi_i (p_1, p_2) = \frac{1}{2} (p_i - c) (\alpha - p_i)}$ **if** $\mathbf{p_i = p_j}$

  $\mathbf{\pi_i (p_1, p_2) = 0}$ **if** $\mathbf{p_i > p_j}$

  where **j** is the other firm **(j = 2 if i = 1, j = 1 if i = 2).**
- Let $\mathbf{p_m}$ denote the value of **p** that maximizes $\mathbf{(p_i - c) (\alpha - p_i)}$.

Bertrand's Duopoly

- **Firm i's best response function**:

  $\mathbf{B_i(p_j) = \{p_i: p_i > p_j\}}$ **if** $\mathbf{p_j < c}$

  $\mathbf{B_i(p_j) = \{p_i: p_i \geq p_j\}}$ **if** $\mathbf{p_j = c}$

  $\mathbf{B_i(p_j) = \emptyset}$**, if** $\mathbf{c < p_j \leq p_m}$

  $\mathbf{B_i(p_j) = \{p_m\}}$ **if** $\mathbf{p_j > p_m}$**.**

Bertrand’s Duopoly

- **Explanation**:
- If $p_j < c$, then firm i’s profit is negative if $p_i \leq p_j$ and zero if $p_i > p_j$. Therefore, any price greater than p_j is a best response to p_j (i.e., $B_i(p_i) = \{p_i: p_i > p_j\}$).
- If $p_j = c$, since p_j as well as any price greater than p_j yields a profit of zero, $B_i(p_j) = \{p_i: p_i \geq p_j\}$.
- If $c < p_j \leq p_m$, assuming that the price can be any number (i.e., a continuous variable), $B_i(p_j) = Ø$ (since firm **i** wants to choose a price less than p_j, but is better off the closer that price is to p_j. For any price less than p_j there is a higher price that is also less than p_j, so there is no best price).

Bertrand’s Duopoly

- If $p_j > p_m$, then p_m is the unique best response of firm **i**, that is, $B_i(p_j) = \{p_m\}$.
- Nash Equilibrium: $(p_1^*, p_2^*) = (c, c)$.
- The game has a single Nash equilibrium, in which each firm charges price **c**.
- Conclusion: When the unit cost of production is a constant c, the same for both firms, and demand is linear, Bertrand’s game has a unique NE, in which each firm’s price is equal to **c**.

Comparison between the Outcomes of Cournot’s Game and Bertrand’s Game

- For a duopoly in which both firms have the same constant unit cost and the demand function is linear, NE of Cournot’s and Bertrand’s games generate different economic outcomes.
- The equilibrium price in Bertrand’s game is equal to the common unit cost **c**, whereas the price associated with the equilibrium of Cournot’s game is $1/3\ (\alpha + 2c)$, which exceeds **c** because $c < \alpha$.

Stackelberg’s Model of Duopoly [SM30]

- In Stackelberg competition it is assumed that at least one of the firms in the market is able to precommit itself to a particular level of supply before other forms have fixed their level of supply.
- Other firms observe the leader’s supply and then respond with their output decision.
- The firms able to initially precommit their level of output are called the market leaders and the other firms are the followers.

Stackelberg’s Duopoly

- Consider a market in which there are two firms, both producing the same good.
- Firm **i**’s cost of producing q_i units of the good is $c_i(q_i)$; the price at which the output is sold when the total output is **Q** is **P(Q).**
- Each firm’s strategic variable is output (as in Cournot’s model), but the firms make their decisions sequentially, rather than simultaneously: one firm chooses its output, then the other firm does so, knowing the output chosen by the first firm.
- Firm 1 chooses a quantity $q_1 \geq 0$, and Firm 2 observes q_1 and then chooses q_2. The resulting payoff or profit for firm **i** is

$$\pi_i(q_1, q_2) = q_i\ (P(Q) - c_i)$$

where $Q = q_1 + q_2$, and $P(Q) = \alpha - Q$ is the market clearing price when the total output in the market is **Q**.

Stackelberg’s Duopoly

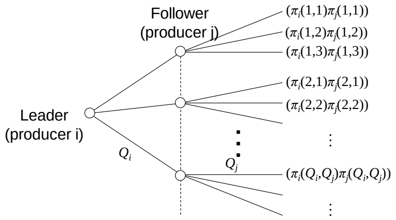
- The game is a two-person sequential game with two stages and with perfect information.
- Use the backward induction method to solve the game: find reaction of Firm 2 to every output choice of Firm 1. Hence find output q_2^* that maximizes Firm 2’s profit given output q_1. That is, $q_2^* = q_2^*(q_1)$ solves
- $\pi_2(q_1, q_2^*) = \max \pi_2(q_1, q_2) = \max q_2(\alpha - q_1 - q_2 - c_2)$, **subject to** $q_2 \geq 0$.
- Taking the first derivative with respect to q_2 and equating it to zero gives

$q_2^* = q_2^*(q_1) = (\alpha - q_1 - c_2)/2$, provided $q_1 < \alpha - c_2$.

Stackelberg’s Duopoly

- Firm 1 should now anticipate that Firm 2 will choose $\mathbf{q_2^*}$ if Firm 1 chooses $\mathbf{q_1}$. Therefore, Firm 1 will want to choose $\mathbf{q_1}$ to maximize the function
- $\boldsymbol{\pi_1(q_1, q_2^*) = q_1(\alpha - q_1 - q_2^* - c_1) = ½ [-q_1^2 + (\alpha + c_2 - 2c_1)q_1]}$ **subject to** $\mathbf{q_1 \geq 0}$**.**
- Subsequently, the maximizer of $\boldsymbol{\pi_1(q_1, q_2^*)}$ is obtained as $\boldsymbol{q_1^* = ½ (\alpha + c_2 - 2c_1)}$**,** which is the equilibrium strategy of firm 1.
- Therefore, $\boldsymbol{q_2^* = ¼ (\alpha + 2c_1 - 3c_2)}$**,** which is the equilibrium strategy of Firm 2.
- If $\boldsymbol{c_1 = c_2, q_1^* = ½ (\alpha - c)}$**,** and $\boldsymbol{q_2^* = ¼ (\alpha - c)}$**.** Therefore, Firm 1’s profit is = $\boldsymbol{q_1^*(P(Q) - c) = 1/8 (\alpha - c)^2}$, and Firm 2’s profit is $\boldsymbol{1/16 (\alpha - c)^2}$.

Extensive Form for the Stackelberg Game



Cournot NE vs. Stackelberg NE

- Compared to the Cournot NE, the Stackelberg NE entails higher profits for the leader and smaller profits for the follower.
- The ability of the leader to precommit itself to a particular level of supply has made that firm better off (first-move advantage).

Application of Oligopoly Models for Spectrum Management in Cognitive Radio Networks

- **Cournot Model for Dynamic Spectrum Allocation**
- **Bertrand Game Model for Spectrum Pricing**
- **Stackelberg Game for Optimal Pricing and Bandwidth Sharing**

Cournot Game

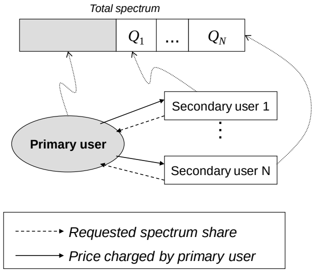
Companies compete on the amount of output they will produce, which they decide on independently of each other and at the same time.

- There is more than one firm and all firms produce a homogeneous product, i.e., there is no product differentiation;
- Firms do not cooperate, i.e. there is no collusion;
- Firms have market power, i.e. each firm's output decision affects the good's price;
- The number of firms is fixed;
- Firms compete in quantities, and choose quantities simultaneously;
- The firms are economically rational and act strategically, usually seeking to maximize profit given their competitors' decisions.

Cournot Game Model for Dynamic Spectrum Allocation among Multiple Secondary Users [SM9]

- The problem of spectrum sharing among the primary user and multiple secondary users can be formulated as an oligopoly market competition.
- A Cournot game model is presented for the case where each of the secondary users can completely observe the strategies and the payoffs of other secondary users.
- Objective of this spectrum sharing is to maximize profit of secondary users by utilizing the concept of equilibrium.

Cournot Game Model for Spectrum Sharing



Cournot Game Formulation

- Players: the secondary users
- Strategies: spectrum size (i.e., $\mathbf{Q_i}$ for secondary user i)
- Payoffs: profit (i.e., revenue minus cost) of secondary user **i**
- Nash equilibrium (i.e., $\mathbf{Q^*} = \mathbf{\{Q_1^*, \ldots, Q_N^*\}}$) is considered as the solution of the game which is obtained by the best response function.
- $\mathbf{Q_i^* = BR_i(Q_{-i}^*)}$, for all **i**

Cournot Game: Observations

- NE is located at the point where the best response functions of the secondary users intersect.
- NE depends on relative channel quality among users - with better channel quality, a secondary user may ask for larger spectrum size
- When a secondary user requires larger amount of spectrum, price of spectrum sharing becomes higher, and consequently, allocated spectrum size to other secondary users may become smaller.

Bertrand Game

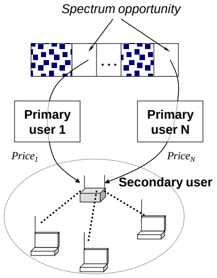
It describes interactions among firms (sellers) that set prices and their customers (buyers) that choose quantities at that price.

- There are at least two firms producing homogeneous (undifferentiated) products;
- Firms do not cooperate;
- Firms compete by setting prices simultaneously;
- Consumers buy everything from a firm with a lower price. If all firms charge the same price, consumers randomly select among them.

Bertrand Game Model for Spectrum Pricing [SM22]

- Pricing is one of the most important issues for dynamic spectrum sharing in cognitive radio networks in order to control resource sharing.
- Consider a competitive situation where a few primary users offer spectrum access to a secondary user. This is formulated as a Bertrand competition in which few firms compete with each other in terms of price to gain the highest profit.
- For a primary user, the cost of sharing the frequency spectrum is modeled as a function of QoS degradation. For the secondary users, a demand function is established based on the utility function which depends on the channel quality.

Bertrand Game Model for Spectrum Sharing and Pricing



Bertrand Model for Spectrum Sharing and Pricing

- Spectrum demand depends on transmission rate in the allocated spectrum and spectrum price (if the spectrum creates higher utility, the demand will be higher).
- Demand function defines the size of the shared spectrum that maximizes the utility of the secondary user given the price offered by the primary user.
- A secondary user can switch among the operating frequency spectrum offered by different primary users (spectrum substitutability).

Bertrand Game

- Players: primary users
- Strategies: strategy of each player is the price per unit of spectrum, which is non-negative
- Payoffs: payoff for each player is the profit of primary user **i** in selling spectrum to the secondary user.
- NE is considered as the solution - obtained by using best response functions

Bertrand Game: Observations

- Consider a cognitive radio environment with two primary service providers and one secondary user (or a set of secondary users controlled by a BS/AP).
- When the first primary service provider increases price, spectrum demand decreases (since the utility decreases). Consequently, cost for the primary user decreases (since size of the spectrum available for primer users increases).
- There is an optimal price for which the profit is maximized (this price is the best response of the corresponding primary user).

Bertrand Game: Observations

- Channel quality of spectrum offered by one service provider impacts the strategies adopted by other service providers.
- When spectrum demand changes for one service provider, other service provider must adapt the price to gain the highest profit.
- Price and profit at NE is higher with better channel quality
- When the secondary user has more freedom to switch among service providers, the level of competition among primary providers becomes higher - therefore, price offered to the secondary user decreases.

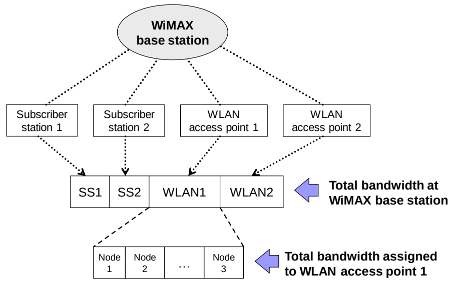
Stackelberg Game Model for Optimal Pricing and Bandwidth Sharing Under Elastic Demand [SM23]

- Spectrum sharing problem between a primary user and multiple secondary users can be also modeled by using a Stackelberg game.
- Objective is to maximize the payoff of the service provider (leader)
- The payoff considers price-elastic and time-varying bandwidth demand of secondary users.
- All the followers choose their best responses given the strategy of the leader.

Example: An Integrated WiMAX/WiFi Network

- An integrated WiMAX/WiFi network where the WiMAX BSs and the WiFi APs/routers are operated by different service providers.
- WiMAX BS charges the WiFi APs/routers for sharing the licensed WiMAX spectrum to provide mobile broadband Internet access to WiFi clients.
- Each AP/router has a dual radio transceiver which can work using both 802.11 and WiMAX interfaces.
- Traffic is transmitted from the BS using WiMAX radio interface and relayed through WiFi AP/router using WiFi interface to WiFi nodes.

An Integrated WiMAX/WiFi Network



Integrated WiMAX/WiFi Network

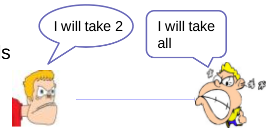
- WiMAX subscriber stations (SSs) have fixed bandwidth demand, and therefore, subscribe at a flat rate to the WiMAX BS.
- WiFi networks have elastic (i.e., time-varying) demand depending on the number of nodes and their preferences.
- Bandwidth demand by a WiFi node depends on the price charged by the WiFi AP/router.
- WiMAX and WiFi service providers have to negotiate with each other to determine the optimal price such that their profits are maximized.

Integrated WiMAX/WiFi Network

- Formulate the pricing problem as a Stackelberg game
  - Players: WiMAX BS (i.e., leader) and WiFi Aps/routers (i.e., followers)
  - Strategies: For the WiMAX BS, the strategy is the price charged to the WiFi APs and for a WiFi AP the strategy is the required bandwidth.
  - Payoffs: For both the WiMAX BS and the WiFi APs/routers, the payoffs are the corresponding profits.
- Obtain the equilibrium of bandwidth sharing and pricing between WiMAX and WiFi service providers.

Spectrum Bargaining Game

- 2 nodes
- 3 Channels



Q: What should be the channel access mechanism?
Sharing rule?
Networks are uncoordinated
Nobody wants to act in an altruistic manner
Networks need to agree on a sharing rule

Issues to consider

- How many channels do each node (player) get?
- Players cannot start communication before deciding on their *share* of channels
  - ☐ How do we capture this issue of patience of the players?
  - ☐ What happens if players go on bargaining indefinitely (infinite horizon) regarding their share of channels?
  - ☐ Is their an equilibrium strategy for the players which maximizes the share of each player against all others?
- More precisely, can we obtain a equilibrium strategy of the players so that they can agree on their share of channels in the *very first* instance?

An example- the *Ultimatum Game*

• 2 hungry players need to divide $10 for buying food

•Patience Factor: Player 1 = 0.6; Player 2 = 0.8
•$10 tomorrow is equivalent to $6 today
•$10 today is equivalent to $16.66 tomorrow

•Discount factor of 1 implies → most patient

Period	P_1's share	P_2'share
0	6.79	3.21
1	5.99	4.01
2	9.99	0.01

TABLE I
Ultimatum Game: The SPNE strategy for the 3-stage game is for P_1 to demand \$6.79 and for P_2 to accept any offer that gives him at least \$3.21

Spectrum Bargaining– More Difficult

- Spectrum can be spatially reused concurrently
  - ☐ Two conflicting players must not use the same channels simultaneously yet well-separated players can.
- Players can only use whole channels

Auction Theory in Dynamic Spectrum Access

Auction

- Auction is a process of buying and selling goods or services through a bidding process. Goods or services are sold to the winning bidders.
- Auction is applied when the price of the goods and services is undetermined and it varies with demand

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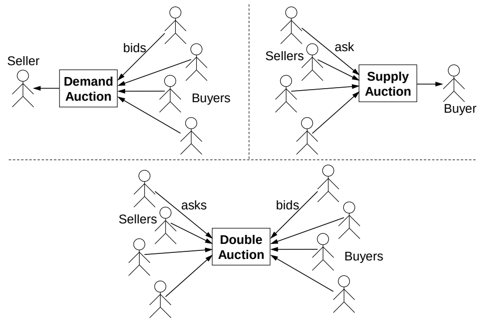
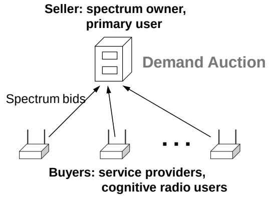
Three Types of Auction



Auction and Spectrum Management

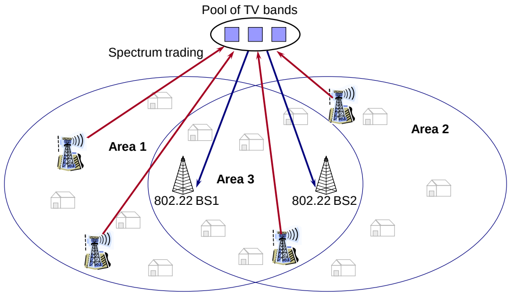
- Theory of auction can be applied to the problem of spectrum trading



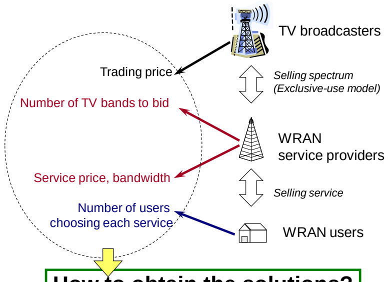
Auction and Spectrum Management

- Joint competitive spectrum bidding and service pricing in IEEE 802.22 networks [SM33]
  - ☐ Sellers: spectrum owner (i.e., TV broadcasters)
  - ☐ Buyers: IEEE 802.22 network service providers

System Model

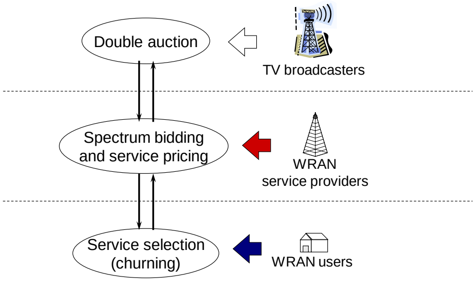


Problem Formulation

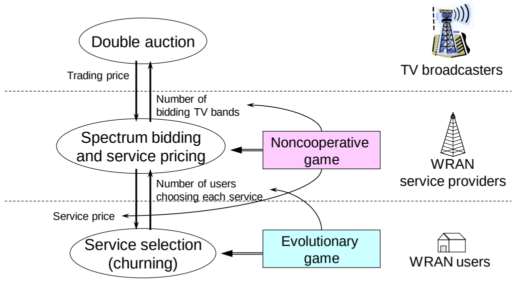


How to obtain the solutions?

Problem Formulation



Game Model Formulations



Game Model Formulations

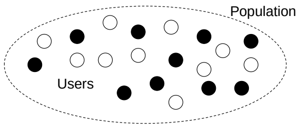
- Evolutionary game model for service selection by WRAN users
  - ☐ Players: WRAN users
  - ☐ Strategies: WRAN service provider
  - ☐ Payoff: Net utility

Payoff Service price

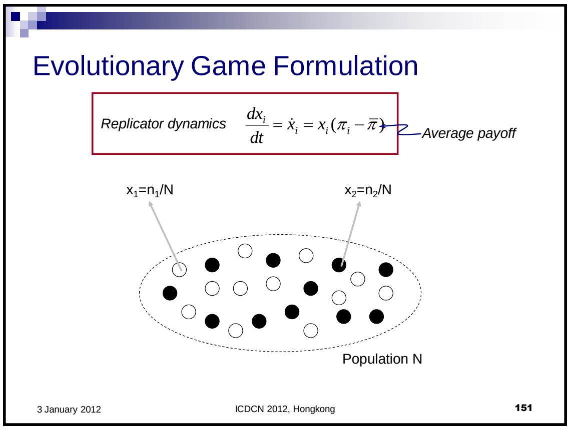
$$\pi_i = U(b_i) - p_i$$

Utility function of allocated bandwidth

Evolutionary Game Formulation

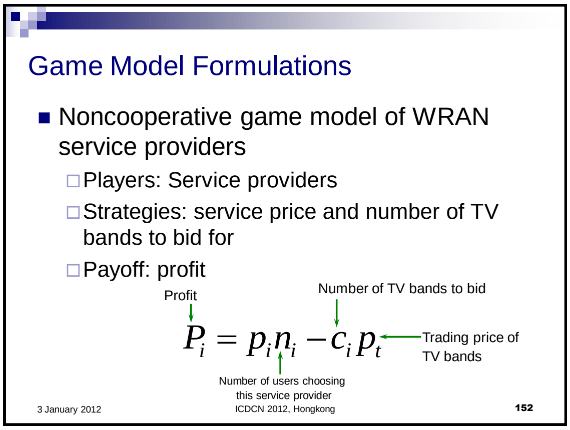


- A user prefers to choose a WRAN service provider with higher payoff in terms of performance (bandwidth) and price
- A user gradually changes the WRAN service provider (i.e., *churns*) by observing the received payoff
- A user does not have any intention to influence the decision of other users in the network



Evolutionary Game Formulation

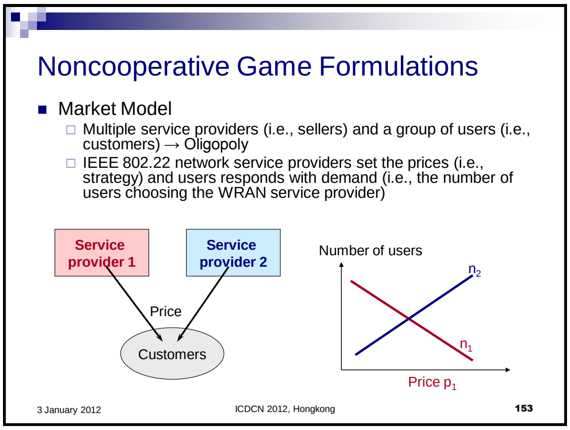
Replicator dynamics $\frac{dx_i}{dt} = \dot{x}_i = x_i(\pi_i - \bar{\pi})$



Game Model Formulations

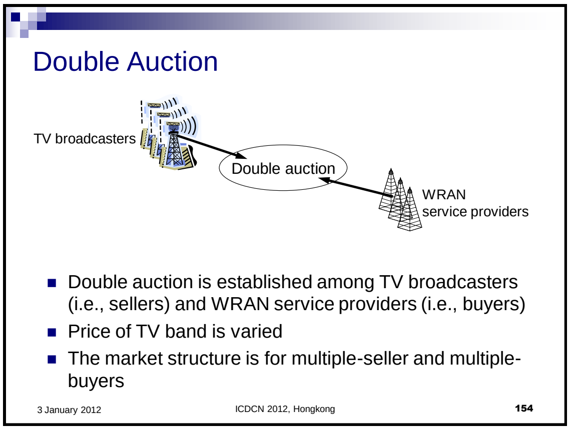
- Noncooperative game model of WRAN service providers
  - ☐ Players: Service providers
  - ☐ Strategies: service price and number of TV bands to bid for
  - ☐ Payoff: profit

$$P_i = p_i n_i - c_i p_t$$



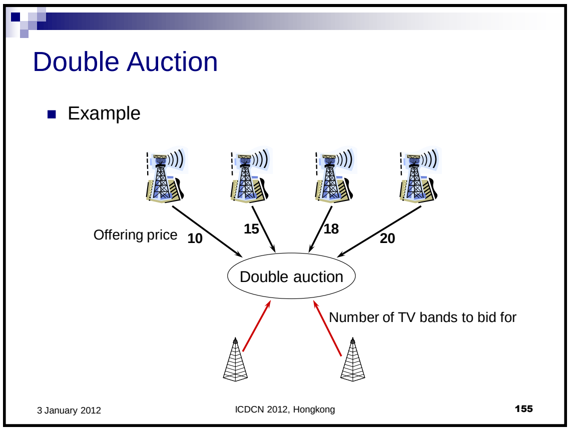
Noncooperative Game Formulations

- Market Model
  - ☐ Multiple service providers (i.e., sellers) and a group of users (i.e., customers) → Oligopoly
  - ☐ IEEE 802.22 network service providers set the prices (i.e., strategy) and users responds with demand (i.e., the number of users choosing the WRAN service provider)



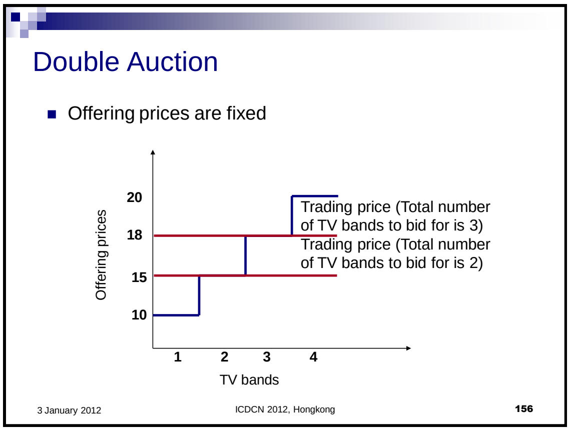
Double Auction

- Double auction is established among TV broadcasters (i.e., sellers) and WRAN service providers (i.e., buyers)
- Price of TV band is varied
- The market structure is for multiple-seller and multiple-buyers



Double Auction

- Example



Double Auction

- Offering prices are fixed

Concluding Remarks

- Cognitive radio based on dynamic spectrum access is a new paradigm for designing wireless communications networks
- Efficient and robust algorithms need to be designed for dynamic spectrum access
- Robust/stable, reliable, and secure protocols need to be developed for communications across distributed and self-configurable cognitive radio networks
  - for dissemination of control traffic
  - for dynamic distributed spectrum access/spectrum mobility management (channel access/bandwidth/channel allocation, pricing, routing, congestion control)
- Regulatory aspects (e.g., rules of cooperation and joint usage between primary and secondary users) need to be addressed
- Hardware/energy constraints/complexity need to be considered for DSA protocol design

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